

The Mathematics of 5771

from Harav Yitzchak Ginsburgh

When considering a number, one of the first analyses we perform on it is looking at its factors, both prime (integers that are divisible only by 1 and themselves¹) and composite (all other numbers).

The fundamental theorem of number theory states that every integer can be expressed as a unique product of prime numbers called its prime factorization.² Therefore, whenever we analyze a number from a purely mathematical point of view, we begin by looking at its prime factors. For example, the prime factorization of 5771 is,

$$29 \cdot 199 = 5771$$

5771 is a special case of a composite number. Because it has no composite factors, it has one and only one factorization into the two prime numbers 29 and 199. As explained in the other article on the secrets of 5771, 29 and 199 are the numerical values of "promise" (הַבְּטָחָה) and "charity" (צְדָקָה), implying that the coming year offers us a unique opportunity to promise and commit to giving charity and to receive the Divine effluence that will allow us to fulfill our promise.

Lucas numbers

However, mathematically, 29 and 199 are related in another important way. They are both Lucas numbers. Elsewhere we have discussed in length additive series, the most important of which is the so-called Fibonacci series, called the Love series in Torah. The first few numbers in the Fibonacci/Love series are:

$$1 \ 1 \ 2 \ 3 \ 5 \ 8 \ 13 \ 21 \ 34 \ 55 \ 89 \ 144 \dots$$

The Lucas series is also an additive series, one in which every number is the sum of the two preceding numbers in the series, except that instead of starting with 1, 1 as does the Love series, Lucas numbers begin with 1, 3, giving the following series of numbers:

$$1 \ 3 \ 4 \ 7 \ 11 \ 18 \ 29 \ 47 \ 76 \ 123 \ 199 \ 322 \ 521 \dots$$

Though there are an infinite number of additive series, the Fibonacci and Lucas series are deeply related mathematically. To see this at the most basic level note how each Lucas number is the sum of two Fibonacci numbers skipping one:



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$$3 = 1 + 2$$

$$4 = 1 + 3$$

$$7 = 2 + 5$$

11 = 3 + 8, and so on.

Additive series are found in various natural phenomena. This is especially true of the Love and Lucas series of numbers. The most famous phenomenon is the hallmark distribution of seeds in the sunflower, as the well as the angle between petals on a rose. Just as for various reasons (beyond our current scope) the Fibonacci series is called the Love series, the Lucas series also has a particularly Jewish interpretation. One way of understanding it is as the nullification or selflessness series of numbers. The numerical value of the Hebrew word for “nullification” or “selflessness” (בטיל) is 47, the 8th number in the series. Moreover, the value of “nullification” in ordinal numbering is 29, the 7th number in the series. The first two letters of “nullification” (בט) equal 11, the 5th number. The final two letters (יל) in ordinal numbering equal 18, the 6th number. Thus, we have 4 consecutive numbers in the series being generated from the word “nullification.”

Since the Lucas/nullification series is found in nature, it follows that when one observes a natural phenomena exhibiting it, one is witnessing the nullification of nature to the Creator. Thus, the coming year of 5771 is especially suited to focusing on how nature nullifies to and therefore reflects the Creator. Observing nature this year with an eye for finding the Creator will be met with surprising insight.

Multiplication of scalars, vectors, and tensors

One of the most useful areas of mathematics in general and discrete mathematics in particular is matrix algebra. Using *gematria* to transform Hebrew words into matrices opens up a completely new area of investigation into the mathematical properties of the Torah. As we will now proceed to see, 5771 can be titled the year of Torah matrix analysis.

The most important word in the Torah is God’s essential four-letter Name, *Havayah* (יהוה).³ Because of its sanctity this Name is never pronounced except for the High Priest during the Yom Kipur service in the Holy Temple in Jerusalem. Instead, we substitute another 4-letter Name of God, *Adni* (אֲדֹנָי)⁴ for it whenever reading it in the Torah or when praying. The Name *Adni* and *Havayah* can be thought of as a vessel and the light contained within the vessel. Since the vessel affects the light, there are many beautiful meditations on the interaction of these two Names.⁵

Now, the most basic mathematical operator (after addition) is multiplication. Applying the multiplication operator to two Hebrew words is straightforward. First, we add the values of the letters in each word to get the sum total of each and then we

multiply the total values of the two words by one another. In this case, the value of *Havayah* (יהוה) is: $\text{י} (10) + \text{ה} (5) + \text{ו} (6) + \text{ה} (5) = 26$, and the value of *Adni* (אדני) is: $\text{א} (1) + \text{ד} (4) + \text{נ} (50) + \text{י} (10) = 65$. Multiplying, we get that the product of *Havayah* and *Adni* is $26 \cdot 65 = 1690$. This is called scalar multiplication and is considered multiplication in 0 dimensions.

But note that by first summing the values of each word's letters and treating each word as a scalar, we have lost resolution on each letter that makes up the original word. In fact, because there is almost always more than one word the sum of whose letters equal the same number, we have no way of distinguishing between them. For example, the word "well" (היטב) in Hebrew has four letters like *Havayah* and its value is 26 also. This means that the product of "well" (היטב) and *Adni* (אדני) is also 1690, just like the product of *Havayah* and *Adni*. Even though there are not an infinite number of words that when multiplied as scalars will yield a particular product, sometimes there can be a few dozen combinations.

More complex than 0-dimensional scalar multiplication is 1-dimensional vector multiplication. Treating each Hebrew word as a vector allows us to retain differentiation between the letters that make up the word. Vector multiplication (sometimes called the dot, or inner product) is a classic operation in Kabbalah and is called "particular multiplication" (הכפאה פרטית). Vector or particular multiplication is limited to when two words have the same number of letters. Thus, in the case of *Adni* (אדני) and *Havayah* (יהוה) carrying out vector multiplication yields:

$$\text{א} \cdot \text{י} + \text{ד} \cdot \text{ה} + \text{נ} \cdot \text{ו} + \text{י} \cdot \text{ה} = 10 + 20 + 300 + 50 = 380$$

Note that the scalar and the vector products of two words are different.⁶

If we would like to write this example in more conventional mathematical notation, it would like this:

$$(1 \ 4 \ 50 \ 10) \cdot (10 \ 5 \ 6 \ 5) = 380$$

Both scalar and vector multiplication are commutative, meaning that it does not matter in what order we multiply the two words. In other words:

$$65 \cdot 26 = 26 \cdot 65$$

And,

$$(1 \ 4 \ 50 \ 10) \cdot (10 \ 5 \ 6 \ 5) = (10 \ 5 \ 6 \ 5) \cdot (1 \ 4 \ 50 \ 10)$$

Vector or particular multiplication is considered a 1-dimensional operation.⁷

When two words have the same number of letters and that number is even, we can exercise the 2-dimensional operator used in matrix multiplication. Let us illustrate using our example of *Adni* and *Havayah*. But, in anticipation of the result, let us write *Adni* in reverse order, i.e., אדני. Writing a word in reverse in Kabbalah

represents the word's reflected light version. Since *Adni* corresponds to the *sefirah* of kingdom, which has no light of its own, but only reflects the light coming to it from the higher *sefirot*, it is actually more meaningful to use its reflected version.

The two matrices we get and their product is thus,

$$\begin{bmatrix} 10 & 50 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 10 & 5 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 400 & 300 \\ 46 & 25 \end{bmatrix}$$

Multiplication of 2 by 2 matrices is usually referred to as tensor multiplication. Whereas there are possibly many positive integer scalars and positive integer vectors that produce the same product when multiplied, one of the results of matrix theory is that if A and B are positive integer matrices (i.e., all of their entries are positive integer numbers) and C is the matrix produced by their multiplication, then there exists no other positive integer matrices whose product is also C (other than the trivial solution CI, where I is the identity matrix)!

5771: Year of the matrix

Now, let us take a closer look at the product of our matrix multiplication of *Adni* and *Havayah*. The sum of the 4 entries in the product is exactly 771!

If the last step, adding the numbers in the resulting matrix seems strange mathematically, just think of it as the reverse of transforming a word into a matrix. If we would substitute letters for numbers in the result we would write:

$$\begin{bmatrix} 400 & 300 \\ 46 & 25 \end{bmatrix} = \begin{bmatrix} ש & ת \\ ה & וּמ \end{bmatrix}$$

Then of course, the sum of these letters—*מו ש ת* and *כה*—is 771.

Writing the resulting matrix out explicitly in this way also reveals that the two diagonals form the words *שמו כהת*, which means “His Name is כהת,” where כהת is the 8th of the 72 three-letter Names of God. The letters כהת are also the initials of the concluding verse of Psalms, “All that has a soul will praise...” (*כָּל הַנְּשָׁמָה תְהַלֵּל*).

3 and 4 dimensions in matrix multiplication

For completeness, let us consider letter filling. Every letter in the Hebrew alphabet has a filling, i.e., the letters that are added to the root letter in order to spell the way the letter is pronounced. Unlike English where the names of the letters are usually meaningless, in Hebrew they form a word that is particularly connected with the letter's meaning and physical shape.⁸

The commonly used letter fillings (some letters can be filled in more than one way) are:

גימל	ג	בית	ב	אלף	א
וו	ו	הא	ה	דלת	ד
טית	ט	חית	ח	זין	ז
למד	ל	כף	כ	יוד	י
סמך	ס	נון	נ	מם	מ
צדיק	צ	פא	פ	עיין	ע
שין	ש	ריש	ר	קוף	ק
				תו	ת

In Kabbalah, letter fillings represent the letter in a more developed or mature state. The filling of the filling (the 2nd filling)—i.e., filling the letter and then filling each of the resulting letters—represents the final and most developed state.

For example, the root, filling, and 2nd filling of the letter א (*alef*) are:

2 nd filling	1 st filling	root
אלף למד פא	אלף	א

Now, we have seen how to convert a word into a matrix. If we would like to convert the word and its first filling, instead of a 2 dimensional matrix we would use a 3 dimensional matrix, where the entries in the 3rd dimension would be populated by the values of the filling letters. To illustrate, the entries in a 2 dimensional matrix are referred to as $a_{i,j}$. The entries in a 2 by 2 matrix are thus,

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$$

The entries in a 3 dimensional matrix are referred to by a triple index, $a_{i,j,k}$. In the case of *Havayah* in a 2 dimensional matrix we write that:

$$a_{1,1} = 10$$

$$a_{1,2} = 5$$

$$a_{2,1} = 6$$

$$a_{2,2} = 5$$

Havayah's first filling is written, יוד הא ואו הא, which we could then write as a 3-dimensional matrix with,

$$a_{1,1,1} = 10; a_{1,1,2} = 6; a_{1,1,3} = 4$$

$$a_{1,2,1} = 5; a_{1,2,2} = 1; a_{1,2,3} = 0$$

$$a_{2,1,1} = 6; a_{2,1,2} = 1; a_{2,1,3} = 6;$$

$$a_{2,2,1} = 5; a_{2,2,2} = 1; a_{2,2,3} = 0$$

For a word and its 2nd filling we would use a 4 dimensional matrix. Illustrating again with *Havayah*, its 2nd filling is יוד ואו דלת הא אלף ואו אלף ואו הא אלף. The resulting matrix representation of this 2nd filling would then be,

$$\begin{aligned} a_{1,1,1,1} &= 10; a_{1,1,1,2} = 6; a_{1,1,1,3} = 4; a_{1,1,2,1} = 6; a_{1,1,2,2} = 1; a_{1,1,2,3} = 6; a_{1,1,3,1} = 4; a_{1,1,3,2} = 30; a_{1,1,3,3} = 400 \\ a_{1,2,1,1} &= 5; a_{1,2,1,2} = 1; a_{1,2,1,3} = 0; a_{1,2,2,1} = 1; a_{1,2,2,2} = 30; a_{1,2,2,3} = 80; a_{1,2,3,1} = 0; a_{1,2,3,2} = 0; a_{1,2,3,3} = 0 \\ a_{2,1,1,1} &= 6; a_{2,1,1,2} = 1; a_{2,1,1,3} = 6; a_{2,1,2,1} = 1; a_{2,1,2,2} = 30; a_{2,1,2,3} = 80; a_{2,1,3,1} = 6; a_{2,1,3,2} = 1; a_{2,1,3,3} = 6 \\ a_{2,2,1,1} &= 5; a_{2,2,1,2} = 1; a_{2,2,1,3} = 0; a_{2,2,2,1} = 1; a_{2,2,2,2} = 30; a_{2,2,2,3} = 80; a_{2,2,3,1} = 0; a_{2,2,3,2} = 0; a_{2,2,3,3} = 0 \end{aligned}$$

1. By convention, we define 1 as a prime number.

2. See Conway and Guy, *The Book of Numbers*, pp. 132-3; Friedberg, *An Adventurer's Guide to Number Theory*, pp. 55-60.

3. See in length in our upcoming book on the number 1820.

4. Note that *Adni* is not the proper pronunciation of this Name either. We do not write the proper pronunciation because of its sanctity, but it is pronounced exactly as written in the Hebrew.

5. Again, see our upcoming book on the number 1820.

6. In fact, given the values of the Hebrew letters, it can be proven that they can never be equal!

7. Perhaps the most beautiful examples of vector multiplication is seen in regard to Adam and Eve. Eve's original name, before the primordial sin was to have been "Chayah" (חַיָּה), which is actually a common Hebrew name; it simply means "living one." When we apply particularized multiplication to Adam (אַדָּם) and Chayah (חַיָּה) we get:

$\aleph \cdot \eta \pm \delta \cdot \gamma \pm \square \cdot \eta = 8 \pm 40 \pm 200 = 248$, the number of prescriptive commandments in the Torah

If we do the same, but this time reverse the letters of Chayah, as is fitting for the feminine which symbolizes the *sefirah* of kingdom which only reflects light (as it has none of its own), we get:

$\aleph \cdot \eta \pm \delta \cdot \gamma \pm \square \cdot \eta = 5 \pm 40 \pm 320 = 365$, the number of prohibitive commandments in the Torah.

Thus, the sum of the particularized multiplication of Adam with Chayah, written forwards and in reverse is 613, the total number of commandments of the Torah!

8. See in length in *The Hebrew Letters*.