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# Torah and Mathematics: Happy Numbers, Singular Numbers, and Family Numbers

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## I. The So-called "Happy Numbers"

Here is a beautiful finding of recreational mathematics.<sup>1</sup> Take any number and sum the squares of its digits. Take the sum and do the same. Repeat the process until you get a number with a single digit. If this number is 1 then the original number is dubbed a "happy" number.

For example, let's take the number 7.

- The square of its only digit, 7, is 49.
- The squares of 4 and 9 are 16 and 81, and their sum is 97.
- The squares of 9 and 7 are 81 and 49, which equal 130.
- The squares of 1 and 3 and 0 are 1, 9, and 0 and their sum is 10.
- The square of 1 is 1 and there is no point in continuing because the square of 1 is 1 and we have an end to the process.

So we have found that 7 goes to 1.

It is easy to show that all other numbers—those that do not go to 1 (and obviously remain there forever)—will end up at 4. But, since the square of 4 is 16 they will then forever cycle through the following numbers: 4, 16, 37, 58, 89, 145, 42, 20, and back to 4, etc.

The origin of this discovery is not entirely clear but it seems to have been documented only in the 20<sup>th</sup> century.

Now the mathematical reason for describing the numbers that end the process at 1 as "happy" is not clear. Even stranger is the idea that numbers that do not end at 1 are "unhappy." According to this terminology, every number is either a happy one or an unhappy four.

Actually, all numbers and, of course, all people should be happy and serve their Creator with joy. Indeed, people that love numbers (the abstract building blocks of



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creation) find joy in the significance of every number. So this choice of terminology does not seem appropriate.

## The Relationship Between 1 and 4

In the Torah, the numbers 1 and 4 are deeply related. Their ratio is considered the most perfect, as it the ratio of words to letters of God's essential Name, *Havayah* (יהוה). For this reason, *Havayah* is called the Tetragrammaton, indicating that it is a name consisting of 4 letters. This ratio is hinted to in the two-letter Biblical word "mist"<sup>2</sup> (אֶד), whose two letters equal 1 and 4 respectively.

One of the models used to relate the four letters of *Havayah* is that of a family comprising a father, mother, son, and daughter, which correspond to the four letters of *Havayah*, in order. Indeed, the minimal requirement for fulfilling the first commandment of the Torah "Be fruitful and multiply," is to bear at least one son and one daughter. Another well-known Kabbalistic model related to *Havayah* in the same way is that of the four Worlds: Emanation (*atzilut*), Creation (*beri'ah*), Formation (*yetzirah*), and Action (*asiyah*).

Foregoing the happy-unhappy nomenclature let us adopt one that is more suited and that carries deeper significance. The number 1, of course, represents singularity, so let us call the numbers that reach one according to the algorithm defined above "singular numbers." Following our identification of the number 4 with the ideal family, let us call the numbers that reach 4, "family numbers." So, all integers are either "singular numbers" (or *sn*, for short) or "family numbers" (or *fn*, for short).

Now let us look at the meaning behind our choice of nomenclature. To be "singular," or "1-oriented," from a spiritual perspective, means that one's consciousness is focused solely on God the Creator, who is absolutely one. To be "family" or "4-oriented" means that one's consciousness is primarily focused on one's mission on Earth, represented by the universal commandment<sup>3</sup> of "be fruitful and multiply," i.e., bearing a family.

## 1 and 4 are "One"

As noted, all *fn* reach the endless cycle of 8 numbers enumerated above. All *sn* reach the forever unchanging 1. Thus, the end of all integers using this algorithm is either 1 or 8 numbers. The 8 numbers that *fn* reach, from 4 to 20, return once more to 4. [In passing let us note that though 4 is of course an *fn*, 44 is an *sn*.]

These three numbers, 1, 4, and 8 carry deep significance in Torah, as they are the values of the three letters that make up the Hebrew word "one" (אחד). This word appears as the climactic termination of the *Shema*, the essential statement of monotheistic

Even more amazingly, if we take the sum of all these numbers: 1 plus 4, 16, 37, 58, 89, 145, 42, and 20—the eight cyclic numbers of the *fn* integers—plus 4 is 416, which is the product of 32 and 13, which are equal to the two Hebrew words “heart” (לב), 32, and “one” (אחד), 13! So, 416 is the product of “one” “heart.”

The number 13 (“one”) is itself the fourth (4) singular (1) number (following 1, 7, and 10)! In addition, the sum of the digits of 13 is 4.

## Singular Numbers and Family Numbers from 1 to 100

Obviously, all powers of 10 are *sn*.

From 1 to 100 there are 20 *sn* and 80 *fn*, which gives a ratio of 1 to 4, or put another way, on the average, 1 out of every 5 numbers from 1 to 100 is an *sn*.<sup>5</sup> This relationship of 20 *sn* in 100 numbers is alluded to in Hebrew by the letter *kaf* (כ) whose numerical value is 20, but whose name “*kaf*,” (כף) when written out, equals 100. In fact, the two letters that spell its name “*kaf*” are *kaf* (כ) and *pei* (פ) whose numerical values are 20 and 80, respectively, alluding to the 20 *sn* and 80 *fn* from 1 to 100!

The 20 *sn* from 1 to 100 are: 1, 7, 10, 13, 19, 23, 28, 31, 32, 44, 49, 68, 70, 79, 82, 86, 91, 94, 97, and 100. Their sum is  $1024 = 32^2$  (note that 32 itself is the 9<sup>th</sup> *sn*,  $9 = 3^2$ ). 32 is also equal to the word “heart” (לב) that we saw above. Since  $32 = 2^5$ , it follows that  $1024 = 2^{10}$ ! Amazingly, 1024 is the number of letters in the entire *Shema* prayer (the expression of “singular” consciousness) that we recite twice a day.<sup>6</sup>

## Singular Numbers and Creation

Now let us contemplate the progression of the number 7 (an *sn*, as we recall) as we apply the algorithm to it. Recall that the progression in this case is 7, 49, 97, 130, 10, 1. The sum of the 6 numbers in this progression is 294. Since  $294 = 6 \cdot 49$ , it follows that the average value of each number in the progression is 49 or  $7^2$ , which is also the second number in the progression. 49 is also the midpoint of 97, the third number of the progression.

The triangle of 7 (i.e., the sum of integers from 1 to 7) is 28 and it too is an *sn*. This relationship is found in the Bible’s first verse: “In the beginning, God created the heavens and the earth,”<sup>7</sup> which in the original Hebrew has 7 words and 28 letters.<sup>8</sup> The

*sn* progression of 28: 28, 68, 100, 1. The sum of the first three numbers 28, 68, and 100 is  $196 = 14^2 = 7 \cdot 28$ . Adding the 1 at the end, the sum of all 4 numbers is 197. The relationship of 28 to 197 is significant in the Torah. As noted, the Bible's first verse, which is the first verse of the account of the first day of creation has 28 letters, while the complete account of the first day (5 verses in all<sup>9</sup>) has 197 letters.

Looking at the composition of the first verse, we find that the *gematria* of the first word, **בְּרֵאשִׁית**, which translates as “In the beginning” is 913, which is also an *sn*!

The *gematria* of the second word, **בְּרָא**, which means “created,” is 203—another *sn*! The same is true of the third word **אֱלֹהִים** (“God”) whose *gematria* is 86, an *sn*!

The *gematria* of the remaining four words in the first verse—“...the heavens and the earth”—are all *fn*. The meaning of this finding is quite clear. The subject matter and hence orientation of the first three words of the Bible, “In the beginning God created” is God as Creator, and all numerical values of all three words are singular numbers. But, the subject matter and hence the orientation of the 4 final words of the first verse, “...the heavens and the earth,” is creation itself, thus they are mission, or purpose oriented. It follows that the *gematria* of these 4 words would be family numbers.

### Inter-inclusion of Singular and Family Numbers

Perhaps the most surprising thing about singular numbers is that 26, the value of God's essential Name, is not one. But, when we look at *Havayah*, letter by letter, we find that the *gematria* of the first letter, *yud* (י), is 10, a singular number. But, the numerical values of the remaining three letters, *hei*, *vav*, and *hei* (היה), are family numbers. Another way of denoting singular and family numbers is “ones” and “fours.” If we apply this nomenclature to the letters of *Havayah* then we have 1 “one” and 3 “fours,” or altogether  $1 \cdot “1” + 3 \cdot “4” = 13$ , an *sn*, which as we recall is the value of the word “one” (אחד), in Hebrew (and the word following *Havayah* in the *Shema*).

At the same time, 86, the *gematria* of Elokim—God's Name that indicates His many faceted nature (we can call this His “fourness”)—is an *sn*! This illustrates that in spite of God's multi-faceted nature, He is one. Additionally it shows us that there is a strong element of inter-inclusion between singular and family numbers.

Another example of this inter-inclusion is the number 70, which is the 13<sup>th</sup> *sn*. In Torah, the number 70 represents the number of “faces” or possible interpretative “perspectives” with which the Torah can be understood, indicating its multi-faceted nature too. Of course, as we already know, 13 is the value of “one” alluding again to the fact that regardless of its multi-faceted nature, the Torah is one.

In Kabbalah, the rectification of male and female lies in their inter-inclusion: the male needs to connect with his feminine aspect while the female needs to express her masculine side. Now, 97 is the 19<sup>th</sup> *sn*. 19 is the gematria of “Eve” (חַוָּה), the first woman. 97 is the gematria of *Meheitavel* (מְהֵיטָבֵאל) a somewhat mysterious feminine figure, who is considered to be the ultimate rectified female. *Meheitavel*’s unique rectified nature is related to the fact that 97, the value of her name, is the sum of 45 and 52, the two numbers that represent the essential masculine and feminine nature within *Havayah*,<sup>10</sup> God’s essential Name.<sup>11</sup> In fact, 45 is the *gematria* of “Adam” (אָדָם), the first male. We mentioned above that singular numbers have a Divine orientation that can be described as a focus on one’s Divine source while family numbers have a mission-oriented approach that can be described as a focus on one’s mission in life. Indeed, in Chassidic teachings it is explained that the feminine attains rectification by focusing upon God while the masculine attains rectification by focusing upon its earthly mission.<sup>12</sup> Thus, “Eve” (19) is a singular number, while “Adam” (45) is a family number.

One final example of the relationship between singular and family numbers can be found in the numbers associated with the commandments. The Torah has 365 prohibitive commandments and 248 positive commandments. 365 is a singular number,<sup>13</sup> while 248 is a family number. We can understand this by characterizing prohibitive commandments as a “turning away” from the world and focusing upward upon God, while positive commandments represent a mission-in-world consciousness.

Now, an obvious rule for *sn* and *fn* is that all permutations of the digits of a number of either type are of the same type. Thus, since 365 is an *sn*, it follows that 356, 653, 635, 536, and 563 are all *sn* also. In our present context, the most interesting of these permutations is the fifth (in the Kabbalistic order of enumerating permutations), 536, which is the value of the Hebrew word “commandments” (מִצְוֹת). If we do the same for 248, we find that the fifth of its permutations, 824, is equal to the well-known idiom “the joy of a mitzvah” (שְׂמֵחָה שֶׁל מִצְוָה). So, the prohibitions are related to the commandments in and of themselves, while the prescriptive positive commandments are related to the joy received from performing a commandment of the Almighty. Using Chassidic terminology, we would say that the prohibitive commandments highlight the feeling of nullification (before God) needed to perform a commandment,<sup>14</sup> while the prescriptive commandments highlight the feeling of joy received from the performance of the commandments. Nullification is the inner motivating force of the *sefirah* of wisdom; joy is the same for understanding. As explained in Kabbalah, the origin of the prohibitive commandments is indeed in wisdom, while the origin of the prescriptive

Incidentally, 613, the total number of commandments is an *fn*, indicating that the ultimate purpose of all the *mitzvot* is to engage in mundane reality and rectify it, first and foremost through the nurturing of a family. Indeed, adding 613 to 248 (both *fn*, as noted), we get 861, the triangle of 41 (the sum of integers from 1 to 41), where 41 is the *gematria* of “mother” (אם).<sup>15</sup>

## Singular and Family Numbers in the Hebrew Alphabet

The Hebrew alphabet contains 22 letters (27 including terminal forms). Every letter has an assigned value, as follows:

<i>alef</i>	א	1	<i>yud</i>	י	10	<i>kuf</i>	ק	100
<i>bet</i>	ב	2	<i>kaf</i>	כ	20	<i>reish</i>	ר	200
<i>gimel</i>	ג	3	<i>lamed</i>	ל	30	<i>shin</i>	ש	300
<i>dalet</i>	ד	4	<i>mem</i>	מ	40	<i>tav</i>	ת	400
<i>hei</i>	ה	5	<i>nun</i>	נ	50			
<i>vav</i>	ו	6	<i>samech</i>	ס	60			
<i>zayin</i>	ז	7	<i>ayin</i>	ע	70			
<i>chet</i>	ח	8	<i>pei</i>	פ	80			
<i>tet</i>	ט	9	<i>tzadik</i>	צ	90			

Contemplating these values, we see that of the 22 letters, only 5 are *sn*:

- *alef* (א) = 1
- *zayin* (ז) = 7
- *yud* (י) = 10
- *ayin* (ע) = 70
- *kuf* (ק) = 100

The remaining 17 letters are all *fn*. The sum of the *sn* letters is 188, which is also an *sn*, is a beautiful example of self-reference!

But more importantly, using the language derived above, we now have 5 “ones” and 17 “fours” in the alphabet, which gives us  $5 \cdot “1” + 17 \cdot “4” = “73”$ . 73 is the 22<sup>nd</sup> (the total number of letters in the alphabet) prime number, another beautiful example of self-reference.

73 is also the numerical value of “wisdom” (חכמה), in Hebrew, alluding to the verse, “You [God] have created all with wisdom,”<sup>16</sup> which can now be interpreted as God having created all with the 22 letters of the Hebrew alphabet, which divide into 5 “ones” and 17 “fours,” totaling 73, “wisdom.”<sup>17</sup>

The sum of the *gematria's* of all 22 letters is 1495. If we add our “73” to this, we get 1568, which is  $2 \cdot 28^2$ , where 28 is the numerical value of the word “strength” (כֹּחַ), alluding to another verse describing the purpose of creation: “The strength of His actions He described to His people.”<sup>18</sup> 1568 is also the *gematria* of the verse, “In all your ways you shall know Him and He will straighten your paths,”<sup>19</sup> which alludes to natural Divine consciousness, the final rectified state of mind of our present reality.<sup>20</sup>

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<sup>1</sup>. Galileo once said that nature is a book and it is written with mathematics. Indeed we find that the Book of Formation, the first Kabbalistic text begins by stating that, “With three books the Almighty created the world...,” and that the language of these books is both the 22 letters of the Hebrew alphabet and the 10 *sefirot*, which are also the foundation of the integers, the building blocks of mathematics. Still, we might think that recreational mathematics are not part of the text of creation, yet even in English, the word “recreational” contains “creation,” hinting that though recreational mathematics do not directly apply to the definition of the laws of physical creation, they do inspire acts of creation (continuous creation— re-creation).

Kabbalah describes that before creating the world, the Almighty engaged in “recreation” (שְׂעוּשׂוּעִים עֲצָמִים), meaning that He took pleasure in “playing,” as it were, with the what would subsequently become the building blocks of reality (numbers and letters). We see the same process occurring in the creative process of great artists, thinkers, and engineers: to receive inspiration, they first engage in free play and wandering thoughts about whatever comes to mind. It would seem that this state is totally self-involved and would never lead to any kind of useful activity, but the fact of the matter is that it provides the inspirational backdrop for the truly great pieces of art, thought, or engineering that these geniuses produce. Foreshadowing the nomenclature we are going to develop in the present article, the purely solitary state can be compared to the singular numbers. The latent potential for creativity and action in the world (beginning from the act of creating the world itself) within that “recreational” state can be compared to the family numbers.

<sup>2</sup>. Genesis 2:6.

<sup>3</sup>. This commandment is considered universal by many legal authorities because God addressed it to Adam, the first human. Still, it is not considered one of the seven Noahide Laws, and for that reason, other legal authorities learn that non-Jews should also have a family from a verse in Isaiah 45:18 stating that God did not create the world to be uninhabited, but rather to be populated.

<sup>4</sup>. Deuteronomy 6:6.

<sup>5</sup>. The exact ratio of *sn* to *fn* for all integers is not known, but it appears to be approximately 1 to 6. Again, this means that on the average, 1 out of every 7 integers is an *sn*. Building on our analysis in note 1, we can now say that this ratio represents the ratio of the 6 weekdays—devoted to action—to the Shabbat—God oriented and providing the weekdays with inspiration, as the sages state that the Shabbat provides blessing for the weekdays. Indeed, our consciousness on Shabbat is singular, oriented towards the Divine, while our weekday consciousness focuses on mundane reality.

Shabbat is the secret of 7, which itself is the first *sn* after 1. As explained in length in our upcoming volume on the mathematics of Genesis, the Shabbat series of numbers is defined by the

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equation:  $O_n = 6\Delta_n + 1$ , where  $O_n$  designates the  $n^{\text{th}}$  Shabbat number and  $\Delta_n$  designates the triangle of  $n$  (i.e., the sum of integers from 1 to  $n$ ). As can readily be seen, Shabbat numbers take the form of hexagons (in other mathematics texts, they are called hexagonal numbers). The first two Shabbat numbers are 1 and 7; the third is 19. 19 is the 5<sup>th</sup>  $sn$  (after 13). The 6<sup>th</sup> Shabbat number is 91, the 17<sup>th</sup>  $sn$ . After 91 come three consecutive Shabbat numbers that are all  $sn$ —331, 397, 469, the 11<sup>th</sup>, 12<sup>th</sup>, and 13<sup>th</sup> Shabbat numbers respectively. 469, the 13<sup>th</sup> Shabbat number and the 7<sup>th</sup> Shabbat number that is also a  $sn$  (and a multiple of 7 as well), is the number of words in the first account of creation in the Torah, from day 1 to day 7, Shabbat, inclusive.

1  $sn$  for every 6  $fn$  can be written symbolically as  $1 \cdot "1" + 6 \cdot "4" = "25"$  or  $5^2$ , the number of letters in the first verse of the *Shema*. The *Shema* itself is replete with square numbers, which in the Torah represent a state of perfect inter-inclusion and completeness. The second verse (added only in the liturgy, but not in the Torah) has 24 letters. So the first two verses together have  $49 = 7^2$  letters. The first verse together with the second verse (as in the Torah) has  $64 = 8^2$  letters. Altogether, the *Shema* has  $1024 = 32^2$ .

<sup>6</sup>. If we were to expand the progression for each  $sn$  from 1 to 100, the sum of all the resulting numbers in these 20 progressions would be 3526, or the product of 41 and 86, where 86 is the *gematria* of *Elokim* (אֱלֹהִים) and 41 is the ordinal value of *Elokim* ( $1 + 12 + 5 + 10 + 13$ ).

<sup>7</sup>. Genesis 1:1.

<sup>8</sup>. There are many examples of triangular numbers, which represent evolutionary progress, in the Bible's first verse, as well as in other aspects of the account of creation. For more on triangular numbers see our upcoming volume on the mathematics of Genesis.

<sup>9</sup>. Genesis 1:1-5.

<sup>10</sup>. Mathematically, 97—the *gematria* of *Meheitavel*—is the 26<sup>th</sup> prime number, where 26 is of course the *gematria* of *Havayah*!

<sup>11</sup>. *Meheitavel* was the wife of the eighth Edomite king, *Hadar*, who in Kabbalah represents the beginning of rectification following the shattering of the vessels. The most important element in attaining a rectified state for the male is the ability to wed and live happily with his spouse.

<sup>12</sup>. For this reason, during procreation the woman looks upward while the man looks downward.

<sup>13</sup>. Note that the next number after 365 in its progression is  $3^2 + 6^2 + 5^2 = 70$ .

<sup>14</sup>. Numerically, 365 is a multiple of 73, the value of "wisdom" (חכמה), in Hebrew.

<sup>15</sup>. There is a particular phrase in Psalms (64:7) "From a man's inside and from the depth of his heart" (קָרַב אִישׁ וְלֵב עֲמֹק), where the first part (קָרַב אִישׁ) equals 613 and the second part (וְלֵב עֲמֹק) equals 248!

<sup>16</sup>. Psalms 104:24.

<sup>17</sup>. The 22 days from the 17<sup>th</sup> of *Tamuz* to 9<sup>th</sup> day of *Av* (*Tisha B'av*), otherwise known as the Three Weeks divide into 17 weekdays and 5 special days (3 *Shabbatot*, 1 day of *Rosh Chodesh*, and *Tisha B'av* itself, which is destined to become a holiday). A similar idea, specifically regarding the Hebrew alphabet is found in the writings of Rabbi Avraham Abulafia. He writes that the 22 letters divide essentially into 17 and 5, the 5 being those letters that have a terminal form.

<sup>18</sup>. Psalms 111:6. See also the first *Rashi* on the Torah.

<sup>19</sup>. Proverbs 3:6.

<sup>20</sup>. See in length in our Hebrew volumes, *Mooda'oot Tivit* and *Hateva Hayehudi*.